Module 3: Linear and Exponential Relations
Overview

Please be advised this is just a tentative outline for Module 3. The order of teaching will need to be revised when the state releases common vocabulary, lessons and assessments.

This is just the standards and topics that will be covered in Module 3.

Pre-Requisites Skills Students need:

To add, subtract, multiply and divide.
What a variable and symbols are.
How to model certain words with mathematical symbols
What the opposite operations are of addition, subtraction, multiplication and division.
Know order of operations
Know inequality symbols and what they mean
Know how to graph a simple inequality on a number line (i.e. $x > 4$)
Know the meanings of exponents ($24 = 2 \times 2 \times 2 \times 2$)
Equation substitution
Point-slope graphing
Tables
Inequalities
Solution sets
Tables and graphing with calculator
Solving a system of equation using tables
Students need to know how to solve equations in terms of variables (i.e. x and y)

Key Vocabulary:**Vocab. from text book, not the state**

Systems of Equations	Greater than (or equal to)	Minimum/Maximum
Solution Set	Less than (or equal to)	End Behavior
Curve	Range	Slope
Intersection point	Vertical Line test	Symmetry
Coordinates	Mapping	Algebraic representation
Parallel lines	Continuous	Relative Minimum/Maximum
Coordinate Grid	Discrete	x-intercept, y-intercept
Half-Plane	Relation	roots
Boundary	Input	translation
Strict boundary	Output	Parent function shift
Interval Notation (i.e. -5<x<12)	Function	Recursive
Included vs. not included	Function Notation	Inside/outside tampering
Domain	Family of functions	Subset
Boundary	Rate of Change	Explicit Fibonacci
Viable	Percent growth/decay	Exponential Grow/decay
Solution/intersection point	Interval	Perimeter
Inclusive/Exclusive		Increasing/decreasing
Constraint		

Module Themes:

Module 3 provides a deep study into exponential functions and introduces Function Notation. It also reviews/formalizes linear functions.

Module 3: Linear and Exponential Relations

Below are the standards covered in Module 3. The order/Topics will be determined in the future.

Core Curriculum Standards:

A-REI.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
A-REI.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
A-REI.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A-REI.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
A-REI.11. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, ~~rational, absolute value~~, exponential, and ~~logarithmic functions~~.

A-CED.3-Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different food.

F-IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
F-IF.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F-IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n greater than or equal to 1.
F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
F-IF.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
F-IF.7.a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
F-IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Module 3: Linear and Exponential Relations

F-LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
F-LE.1.a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
F-LE.1.b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
F-LE.1.c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
F-LE.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
F-LE.5. Interpret the parameters in a linear or exponential function in terms of a context.

F-BF.1. Write a function that describes a relationship between two quantities.
F-BF.1.a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
F-BF.3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
Sequences

F-LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
F-LE.1.a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
F-LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Essential Questions:

Why is the intersection of two equations the solution set of a system of equations?
How do you know the solution set of a system of equations (inequalities) is correct?
How do you determine what strategy to use when solving a system of equations?
How do you determine where to shade on a graph when graphing a two-variable inequality?
How do you determine the solution set of an inequality?
How do you determine if you graph a dotted or solid line when graphing inequalities?
How does the context of the problem limit your solution set?
How do you determine if a relationship is a function?
How can you use family of functions to predict how equations will behave graphically?
What are the differences between linear functions and exponential functions?
What is the significance of the key features of a graphical representation of a function?
How can you use a graph to determine the domain, range, intercepts and rate of change?
What is the difference between arithmetic and geometric sequences and how do they relate to functions?

Module 3: Linear and Exponential Relations

Topic	Key Skills	Chapters	Vocabulary
Systems of Equations- Graphically	Define a solution set of a linear and exponential equations as all the points that satisfy the equations	7.1-Solve Linear systems by graphing **Include different types of graphs (ie the solution of a linear and an exponential)	Systems of Equations Solution Set Curve Intersection point Coordinates Parallel lines Coordinate Grid
Systems of Equations- Algebraically	Solve a system of equations graphically and algebraically. Solve a system of equations and check the solution graphically (show that there is one solution, no solutions or infinitely many solutions graphically) Explain the solution of a system of equations is the intersection point on the graph. Match the solution set to the corresponding graph (Example, a graph a parallel lines has no solution) Justify the method used to solve the system of equations Checking the answer of a solution Show that multiplying an equation by a coefficient does not affect the solution of a system	7.1-Solve linear systems by graphing 7.2-Solve linear systems by substitution 7.3-Solving Linear Systems by adding and subtracting 7.4-Solve linear systems by multiplying first 7.5-Solve special types of linear system **Being able to check and explain why the solution of the system is the intersection, matching type of solution (1 solution, no solution or many) to the graph **Identify and using constraints or restrictions within word problems** **Ability to explain, solve, check solution sets**	Systems of Equations Solution Set Intersection point Coordinates Parallel lines
Graphing Inequalities	Graph an inequality and shade correctly Identify the difference between a strict boundary and where a boundary is not included (dotted vs. solid line) Graph two inequalities on a coordinate grid and identify their solution.	6.7-Graph Linear inequalities in two variables 7.6-Solve systems of linear inequalities **Identify and using constraints or restrictions within word problems**	Half-Plane Boundary Strict boundary Interval Notation (i.e. -5<x<12) Included vs. not included Coordinate Grid

Module 3: Linear and Exponential Relations

Writing Equations	Explain why a solution is valid or not Explain what the solution means in a specific context Explain if a solution makes sense of not Identify the correct domain for the solution within the constraints of a word problem Write the correct inequality symbol based on a word problem Solve systems of inequalities and identify the correct domain for the solution in the context of a word problem.	1.4-Write equations and inequalities Word Problems-6.7 and 7.6 4.2-Graph Linear Equations 4.3-Graphing using intercepts 4.5-Slope-Intercept Form 5.1-Write Linear Equations 5.5-*Writing Equations of parallel and perpendicular lines* EXPAND	Domain Boundary Viable Solution/intersection point Inclusive Exclusive Constraint Greater than (or equal to) Less than (or equal to)
Domain and Range	Identify the domain as the x-values Identify the range as the y-values Define a function as mapping one element of the domain to exactly one element of the range (one to one) Write equation in function notation Graph function y=f(x)	1.6-Representing Functions as Rules and Tables 4.7-Graph Linear Functions	Domain Range Function Notation Vertical Line test Mapping Continuous Discrete Relation
Function Notation	Evaluate a function for a given a given value of x, using function notation. Define f(x) and g(x) Define the function for a given set of outputs	4.7-Graph Linear Functions 5.2-Use linear equations in slope-intercept form 5.4-Write linear equations in standard form Supplemental Materials	Function notation Input output
Domain	Write the correct domain for a function Identify an appropriate domain for a function in the context of a word problem.	4.7-Graph Linear Functions-Word Problems 1.6-Representing Functions as Rules and tables-Word Problems Supplemental Material with emphasis on domain	Domain

Module 3: Linear and Exponential Relations

Function Values	Calculate the y-values given x-values of an equation. Identify the graph for an algebraic equation (linear, polynomial, and exponential). Substitute the coordinate (x, y) into y=f(x) and y=g(x) to verify that the point is the solution to the system. Graph functions and find solutions to systems of equations on my calculator. Construct a table of values for any equation (linear, polynomial and exponential) using my calculator	4.7 Graph Linear funtions Pg. 266 #14 – 21 (more examples) 7.1 Solve Linear systems by graphing Pg. 433 #35. #36 (more examples)	Function Function Notation Family of functions
Rate of Change	Calculate the rate of change from an equation. Calculate the rate of change from a graph Calculate the rate of change from a table Calculate the rate of change of an exponential function using percent growth/decay Calculate the rate of change for a given interval Estimate the rate of change from a graph Describe the rate of change in terms of the context of the situation	4.4-Finding the rate of change	Rate of change Percent growth Percent decay Interval

Module 3: Linear and Exponential Relations

Key Features of Graphs	Identify the slope and y-intercept of a linear function Identify minimums and maximums of a function Identify intervals where a function is increasing or decreasing Identify roots of a graph Identify the x and y intercept. Identify symmetries and end behaviors for graphs Sketch the key features of a function Describe the key features of a function **(This focuses all on linear and exponential)**	4.5 Graphing using Slope – Intercept form pg. 270 Big Idea questions (more examples) 9.4 Solve polynomial equations in Factored form 4.3 Graphing using intercepts pg. 272 more examples	Minimum Maximum Increasing Decreasing Roots Symmetries x-intercept y-intercept relative maximum relative minimum end behavior symmetry slope
Graphing and Interpreting linear functions	Graph a linear function and identify the domain, range, intercepts, and rate of change.	4.7-Graph Linear Functions	
Properties of functions	Compare characteristics of two different functions represented in two different forms (i.e. a table vs algebraically given a linear and exponential) Identify parts of a function algebraically, graphically and verbally		Algebraic representation

Module 3: Linear and Exponential Relations

Linear vs. Exponential	Describe the difference between a linear function and an exponential function. Explain the slope of a linear function as the growth of equal differences over an equal interval. Explain growth/decay of an exponential function	4.2 Graph linear equations 10.8 Compare linear, exponential and Quadratic models 8.6 Write and graph exponential Decay functions	Exponential Growth Exponential Decay
Linear Relationships	Identify when situations can be described by linear functions		
Exponential Relationships	Identify when situations can be described by exponential functions		
Comparing growth of functions	Compare and contrast linear growth and exponential growth *to quadratic growth* from a graph or table Explain why exponential models continue to grow/decay more rapidly than linear or *quadratic* models	10.8 Compare linear, Exponential, and Quadratic models 8.5 Write and graph exponential Growth functions 8.6 Write and graph exponential Decay functions	
Parameters of Functions	Find an ordered pair given a linear function or exponential function. Describe the variables in linear functions in terms of a context (y-intercept is the initial value and the slope is the rate of change) Describe the variables in exponential functions in terms of its context		Parameters

Module 3: Linear and Exponential Relations

Shifts of graph	Identify the parent function of a linear or exponential function. Identify the shift of the graph of a linear or exponential function. Graph the shift of a function as translated of the parent function. Explain the effects of the shifts of graphs using the calculator. Compare the parent function to the function that has been shifted. Identify the translation of a function from the graph and write the function algebraically.	4.7 Graphing Linear functions Page 263 Parent function of an exponential function $y = b^x$ Pg. 522 and 533	Translation Inside (internal) tampering Outside(external) tampering Parent function shits
Sequences	Define an arithmetic sequence Define an arithmetic sequence as a linear function. Define a geometric sequence. Define a geometric sequence as an exponential function. Define an arithmetic or geometric sequence as a function, sometimes defined recursively.	Pg. 309 Pg. 359 **Need more resources**	Arithmetic sequence Geometric sequence Recursive Explicit Fibonacci Subset

Module 3: Linear and Exponential Relations

Explicit and Recursive processes	Explain the steps to set up a linear, exponential, or *quadratic* functions Exponential function explicitly from a word problem Construct a linear function explicitly from a word problem Construct a linear function recursively from a word problem *Construct a quadratic function explicitly and recursively from a word problem*.	Pg. 221 #39, 40, 41 Pg. 220 #37 **Will need more word problems**	
Constructing linear and exponential sequences	Write a linear function or an arithmetic sequence from a graph, description or table. Write an exponential function or geometric sequence from a graph, description or table.	Pg. 309 Pg. 359 **More resources needed**	
Linear vs. Exponential Growth	Describe the difference between linear and exponential functions. Explain the slope of a linear function as compared to the growth of equal differences over equal intervals in an arithmetic sequences.	8.5 Write and graph exponential growth functions 4.7 Graph Linear Functions	Exponential growth Exponential decay Common ratio Common difference